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Equilibrium and Non-Equilibrium in a Reversible and Conservative Cellular Automaton

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Abstract

The main goal of this thesis, relies the dynamics of a reversible and conservative cellular automaton Q2R model. Q2R is a automaton that runs on a two-dimensional grid of finite size and is reversible in a physical sense, that is, not only is the automaton rule invertible, but the backward rule reads exactly the same as the forward one. This model is a dynamical variation of the Ising model for ferromagnetism that possesses quite a rich and complex dynamics.

As expected, the Q2R automaton only possesses fixed points and periodic orbits and it has been shown that possesses an energy like quantity, and, at least an extra conserved quantity. Although, the dynamics includes only fixed points and periodic orbits, numerical simulations show that the system exhibits a ferromagnetic phase transition in the large system size limit for a well defined critical energy.

In the present work, we characterize the configuration space, that is composed of a huge number of cycles with exponentially long periods. More precisely, we quantify the probability distribution functions of states in terms of the aforementioned invariants. We show that the dynamics of the system in the phase space appears to be, depending on the energy, a random walk or a Levy flight.

The main contribution of the present thesis is the application of a coarse-graining approach that allows to write a coarse-grained master equation, which characterizes equilibrium and non equilibrium statistical properties, for the Q2R model. Following Nicolis and collaborators, a coarse-graining approach is applied to the time series of the total magnetization, leading to a consistent master equation that governs the macroscopic irreversible dynamics of the Q2R automata. The methodology is replicated for various lattice sizes. In the case of small systems, we show that the master equation leads to a tractable probability transfer matrix of moderate size, which provides a master equation for a coarse-grained probability distribution. The method is validated and some explicit examples are discussed.

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